

## 8.3 Geometric Sequences and Series

**Geometric Sequence**: a sequence in which each term after the first is obtained by **multiplying** the preceding term by a **fixed nonzero constant**

Ex. 2, 6, 18, 54, 162, ...

**Common Ratio**: the amount that is found by **dividing** any term after the first term by the term that directly precedes it.

$$r = \frac{a_{n+1}}{a_n}$$

$r$  : **common ratio** of a sequence

$a_n$  : **the  $n$ th term**, or **general terms**, of a sequence

Ex. Find the common ratio of 6, -12, 24, -48, 96, ...

**\*\*** When the common ratio of a geometric sequence is **negative**, the signs of the terms alternate.

Ex. Write the first five terms of the geometric sequence in which  $a_1 = 24$  and

$$r = \frac{1}{3}.$$

### ❖ The General Term of a Geometric Sequence

**Formula for The  $n$ th term (the General Term) of a Geometric Sequence**

$$a_n = a_1 r^{n-1}$$

Ex. Find  $a_{75}$  of the geometric sequence with  $a_1 = \frac{1}{2}$  and  $r = -2$ .

Ex. (#36) Find the fifth term of a geometric sequence from the given information.

$$a_1 = 16 \text{ and } a_2 = -12$$

Ex. Given the geometric sequence  $12, 6, 3, \frac{3}{2}, \dots$ .

(a) Write a formula for the  $n$ th term of the geometric sequence. (Use the formula above.)

(b) Use the formula for  $a_n$  to find  $a_7$ .

### ❖ The Sum of the First $n$ Terms of a Geometric Sequence

#### Formula for the Sum of the First $n$ Terms of a Geometric Sequence

$$S_n = \frac{a_1(1-r^n)}{1-r}, \quad r \neq 1$$

Ex. Find the sum of the first 11 terms of the geometric sequence:

$$4, -12, 36, -108, \dots$$

Ex. Find  $\sum_{i=1}^7 4(-3)^i$ . Use the formula for the sum of the first  $n$  terms of a geometric sequence.

## ❖ Geometric Series

### Infinite Geometric Series

An infinite sum of the form:  $a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \cdots + a_1 r^{n-1} + \cdots = \sum_{i=1}^{\infty} a_1 (r)^{i-1}$

### Formula for the Sum of an Infinite Geometric Sequence

$$S_{\infty} = \frac{a_1}{1-r}, |r| < 1$$

Ex. Find  $\sum_{i=1}^{\infty} 4(0.3)^{i-1}$

Ex. Find  $3 - 1 + \frac{1}{3} - \frac{1}{9} + \cdots$

Ex. Express the repeating decimal as a fraction in lowest terms.

(a)  $0.\overline{5} = \frac{5}{10} + \frac{5}{100} + \frac{5}{1000} + \cdots$

(b)  $0.\overline{72}$

Ex. The general term of a sequence is given. Determine whether the sequence is arithmetic, geometric, or neither. If the sequence is arithmetic, find the common difference; if it is geometric, find the common ratio.

(a)  $a_n = n - 3$

(b)  $a_n = \left(\frac{1}{2}\right)^n$

(c)  $a_n = n^2 - 3$